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MAGNETIC RESONANCE ELECTROENCEPHALOGRAPHY (MRE): A VECTOR CALCULUS OF RELATED VOLUMES AND SURFACES

Robert W. Thatcher, Ph.D.
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I wrote these equations at the time that myself and colleagues were publishing a series of studies that related the EEG to MRI T2 relaxation time (Thatcher et al, 1977; 1998a; 1998b). A later 2001 publication (Thatcher et al, 2001) presented some of the empirical evidence in support of the equations and the first presentation of the concept of Magnetic Resonance Electroencephalography (MRE) was presented as an abstract in 1999 (Thatcher et al, 1999). The equations represent a marriage between EEG and MRI whereby there is a deeper or fundamental mathematical and physical equivalence between the electrical energies of the EEG and the kinetic energies expressed by a random walk or Brownian motion of atoms as measured by the MRI (Feynmann, 1963; Malmivuo and Plonsey, 1995). I use the standard vector calculus of the electrical dynamics of Clerk Maxwell's 1864 velocity distribution and Simone Poisson's 1810 current density equations as they apply to proton spin coherence (entropy and Brownian movement) and EEG current density. The unification is possible because these processes are empirically related and can only be understood by an equivalence between the macromolecular complexity of small volumes of atoms (1mm^3 voxels) in the cortex and the current source density of the EEG given some constant. The MRI T2 relaxation time is a measure of spatial heterogeneity and entropy that is governed by the Einstein-Boltzmann description of random and Gaussian distributed motions of atoms in a volume. The lower the density of atoms then the longer is the T2 relation time (a cup of water in a MRI machine takes about 3 seconds to relax and a metal takes

microseconds). T2 relaxation time is also related to the amplitude and coherence of the human electroencephalogram and is therefore related to the electrical inductance and capacitance properties of cortical neural membranes (e.g., Hodgkin & Huxley). The linkage between the atomic entropy and the EEG is represented by the density of protons in small volumes in the cortex (Thatcher et al, 2001). The parameter of a wave equation is MRI T2 relaxation time in milliseconds which operates like a band pass filter also seen in resonant neurons. I called the relationship between the Brownian motion of protons and the equations of electricity: Magnetic Resonance Electronencephalography (MRE). After I wrote these equations I moved on to other issues in neuroscience and I did not publish them nor fully perfect them. However, I decided to retrieve these thoughts and share the equations after discussions with my friend Tom Collura who understands how difficult it is to think about a merger between EEG and MRI. These equations are only understandable to those who are trained in the language of vector calculus which is typically advanced physics students. For those that are not versed in vector calculus I recommend reading Richard Feynman's first year college physics text because he is a great teacher. Whether this attempt is successful or not, just thinking about the relationship between the density of atoms and the frequency characteristics of the EEG was a great personal joy and although I may or may not return to this topic in the future it will always be an intellectual gem.

1.0 Vector Field of Proton Relaxation Time (T2).

EQ. 1 $\frac{\partial^2 T_2}{\partial^2 X} + \frac{\partial^2 T_2}{\partial^2 Y} + \frac{\partial^2 T_2}{\partial^2 Z} = \nabla^2_{T2}$ - This is the divergence of T2 relaxation time assuming thermodynamic entropy related to the density of protons per unit volume, i.e., the tendency to become smooth.

1.1 Vector Field of EEG Current Source Density

EQ. 2 $$\frac{\partial^2 J_x}{\partial^2 X} + \frac{\partial^2 J_y}{\partial^2 Y} + \frac{\partial^2 J_Z}{\partial^2 Z} = \nabla^2_{CD}$$ - This is the divergence of EEG current sources, assumes the standard Poisson equation for electricity, i.e., electrical sources.

2.0 Magnetic Resonance EEG as Spatial Vector Fields of Brain T2 Relaxation Time and EEG Current Source Density

EQ. 3 $$\nabla^2_{T2} = 4\Pi\rho_{T2}$$ - Poisson's divergence for T2 relaxation time

EQ. 4 $$\nabla^2_{CD} = 4\Pi\rho_{CD}$$ - Poisson's divergence for EEG

EQ. 5 $$\rho_{T2} = \frac{\nabla^2_{T2}}{4\Pi}$$ - Macromolecular Solution

EQ. 6 $$\rho_{CD} = \frac{\nabla^2_{CD}}{4\Pi}$$ - Electricity Solution

3.0 Magnetic Resonance EEG (MRE) Dot Product of T2 Relaxation Time and EEG Current Source Density

EQ. 7 $$\rho_{MRE} = \rho_{CD} \bullet \rho_{T2}$$ - MRE dot product of Scalars

EQ. 8 $$\nabla^2_{MRE} = \nabla^2_{CD} \bullet \nabla^2_{T2}$$ - MRE dot product of vectors

4.0 Magnetic Resonance EEG (MRE) Curl of T2 Relaxation Time and EEG Current Source Density

EQ. 9 $$\rho_{MRE}^{(curl)} = \rho_{CD} \times \rho_{T2}$$

EQ. 10 $$\nabla^{2(CURL)}_{MRE} = \nabla^2_{CD} \times \nabla^2_{T2}$$

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